11/14/19

MIS10 (Continued)

More Examples:

Multiple-Lives ((x) & (y)) Single Decrement

States: 0: both are alive

1: (x) is still alive but (y) has died

2: (x) has died but (y) is still alive

3: both have died

$$\begin{tikzcd}
(0) \arrow[r, "\mu_{xy}^{01}"] \arrow[d, "\mu_{xy}^{02}"'] \arrow[dr, "\mu_{xy}^{03}"] & (1) \arrow[d, "\mu_{x}^{13}"] \\
(2) \arrow[r, "\mu_{y}^{23}"'] & (3)
\end{tikzcd}$$

Q: Under what condition is it true that (x) & (y) have dependent lifetimes

A: $\mu_{xy}^{03} \neq 0$, or

$\mu_{xy}^{02} \neq \mu_{x}^{13}$, or

$\mu_{xy}^{01} \neq \mu_{y}^{23}$

Single Life Multiple Decrement

<u>States</u>: 0: (x) is "active"

1: (x) has departed by decrement 1

2: ———————————— 2

$$
\begin{tikzcd}
 & (0) \arrow[dl] \arrow[d] \arrow[dr] & \\
(1) & (2) & (3)
\end{tikzcd}
$$

μ_x^{01}, μ_x^{02}, μ_x^{03}

$\mu_x^{23} = 0$

<u>Remarks</u>:

$${}_nP_x^{00} = {}_nP_x^{(\tau)}$$

$${}_nP_x^{01} = {}_nq_x^{(1)}$$

$${}_nP_x^{02} = {}_nq_x^{(2)}$$

$${}_nP_x^{(\tau)} = e^{-\int_0^n \mu_{x+t}^{(\tau)}\,dt} \qquad \mu_{x+t}^{(\tau)} = \sum \mu_{x+t}^{(j)}$$

For multistate models $\mu_x^{01} + \mu_x^{02} + \mu_x^{03} + \cdots = \mu_x^{0\tau}$

total force of transition out of state 0.

$\therefore$ in this case

$${}_nP_x^{00} = e^{-\int_0^n \mu_{x+t}^{0\tau}\,dt}$$

True because in this model, if we ever leave state 0, we cannot <u>return</u> to state 0.

<u>Notation</u>: ${}_nP_x^{\overline{ii}} = Pr($an x-year old in state i remains in state i until age $x+n)$

above $\cancel{{}_nP_{00} = {}_nP_{\overline{00}}}$ ${}_nP_x^{00} = {}_nP_x^{\overline{00}} = e^{-\int_0^n \mu_{x+t}^{0\tau}\,dt}$

Compare, generally, ${}_nP_x^{11}$ to ${}_nP_x^{\overline{11}}$

(x) may not be in state 1 the entire time (~~hard to~~ impossible to calculate exactly)

(x) is in state 1 the entire time (easy to calculate)

$$\boxed{{}_nP_x^{\overline{ii}} = e^{-\int_0^n \mu_{x+t}^{i\bullet}\,dt}}$$

Another example

<u>Permanent</u> Disability Model

States

0: ~~hea~~ (x) is healthy

1: (x) is disabled

2: (x) has died

no transition from (1) to (0)

$$\begin{tikzcd}
(0) \arrow[rr] \arrow[dr] & & (1) \arrow[dl] \\
& (2) &
\end{tikzcd}$$

Probabilities:

$${}_nP_x^{20} = 0 = {}_nP_x^{21} = {}_nP_x^{10} \qquad {}_nP_x^{22} = 1$$

$${}_nP_x^{00} = {}_nP_x^{\overline{00}} = e^{-\int_0^n \mu_{x+t}^{0\bullet}\,dt} \qquad \mu_{x+t}^{0\bullet} = \mu_{x+t}^{01} + \mu_{x+t}^{02}$$

$${}_nP_x^{11} = {}_nP_x^{\overline{11}} = e^{-\int_0^n \mu_{x+t}^{1\bullet}\,dt} \qquad \mu_{x+t}^{1\bullet} = \mu_{x+t}^{12}$$

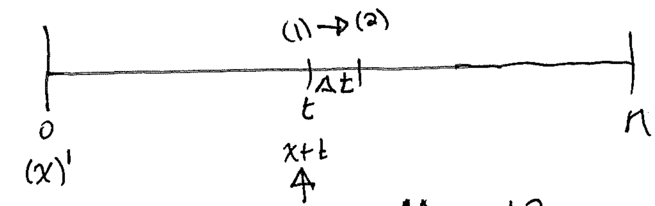
Q: ${}_nP_x^{12} = ?$

Note: $\underbrace{{}_nP_x^{10}}_{=0} + \underbrace{{}_nP_x^{11}}_{= \underbrace{{}_nP_x^{\overline{11}}}_{\text{above}}} + {}_nP_x^{12} = 1$

$\therefore \; {}_nP_x^{12} = 1 - {}_nP_x^{\overline{11}}$ (easy way)

(harder way)

${}_nP_x^{12} = ?$



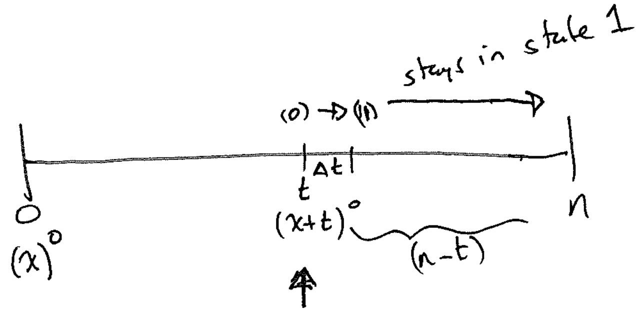
$$Pr = {}_tP_x^{11} \cdot \mu_{x+t}^{12} \cdot \Delta t$$

$$\therefore \; {}_nP_x^{12} = \int_0^n {}_tP_x^{11} \cdot \mu_{x+t}^{12} \, dt \; \underset{\text{model}}{\overset{\text{in this}}{=}} \; \int_0^n {}_tP_x^{\overline{11}} \cdot \mu_{x+t}^{12} \, dt$$

Recall the model:

$$\begin{tikzcd} (0) \arrow[rr] \arrow[dr] & & (1) \arrow[dl] \\ & (2) & \end{tikzcd}$$

${}_nP_x^{01} = ?$



$$Pr = {}_tP_x^{00} \cdot \mu_{x+t}^{01} \cdot \Delta t \cdot {}_{n-t}P_{x+t}^{11}$$

$${}_nP_x^{01} = \int_0^n {}_tP_x^{\overline{00}} \cdot \mu_{x+t}^{01} \cdot {}_{n-t}P_{x+t}^{\overline{11}} \, dt$$